# Using GPS Signals to Measure Electron Density in the Ionosphere

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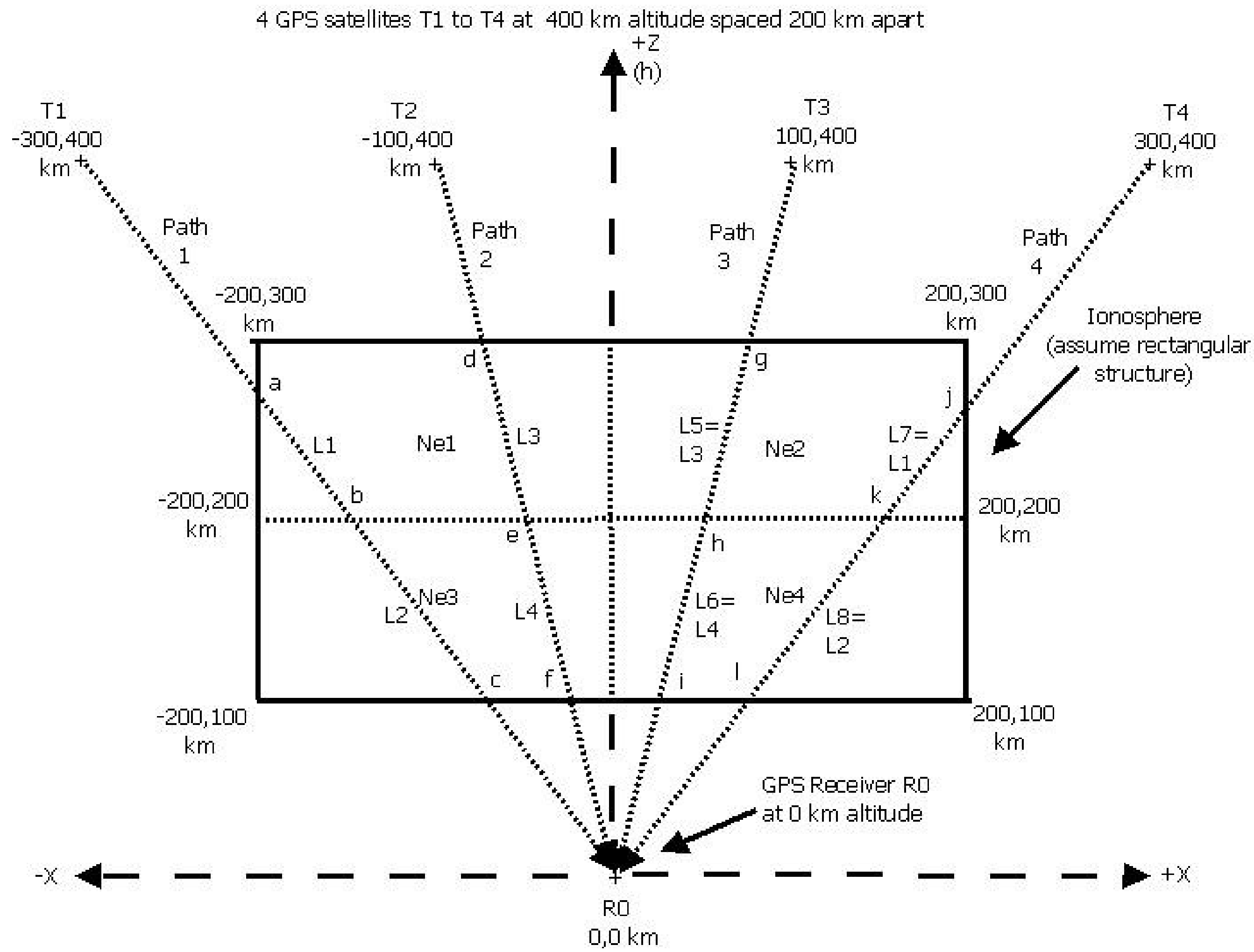
Here's a highly simplified explanation of how electron density can be calculated from measured delays in the GPS signal. Figure 1 shows how measured delays for four signal paths are used to find the electron densities $Ne_1$ – $Ne_4$ in four regions of the ionosphere.

Figure 1: 2-D model of signal paths from four GPS satellites to a ground-based receiver.




Assumptions are:
Geometry for signal paths and ionospheric structure as shown in Figure 1
Signal delay proportional to distance within given Ne for given path
No delay outside the model structure

Delay for each signal path is found by measurements $M_1$ to $M_4$. The total electron content STEC is calculated from the delay measurements:

$M_1 \rightarrow STEC_1 = 4.583 \times 10^{16}$ electrons
$M_2 \rightarrow STEC_2 = 4.124 \times 10^{16}$ electrons
$M_3 \rightarrow STEC_3 = 6.186 \times 10^{16}$ electrons
$M_4 \rightarrow STEC_4 = 6.666 \times 10^{16}$ electrons

Because signal delay is proportional to the length of the path with a given electron density Ne, we obtain:

| | | | | |
|---|---|---|---|---|
| $STEC_1 = L_1 \times Ne_1$ | | $+ L_2 \times Ne_3$ | | (Path 1) |
| $STEC_2 = L_3 \times Ne_1$ | | $+ L_4 \times Ne_3$ | | (Path 2) |
| $STEC_3 =$ | $L_5 \times Ne_2$ | | $+ L_6 \times Ne_4$ | (Path 3) |
| $STEC_4 =$ | $L_7 \times Ne_2$ | | $+ L_8 \times Ne_4$ | (Path 4) |

Using some elementary geometry we can calculate:

$L_1 = L_7 = 83.333$ km
$L_2 = L_8 = 125$ km
$L_3 = L_5 = 103.1$ km
$L_4 = L_6 = 103.1$ km

Converting km to meters and substituting, we obtain:

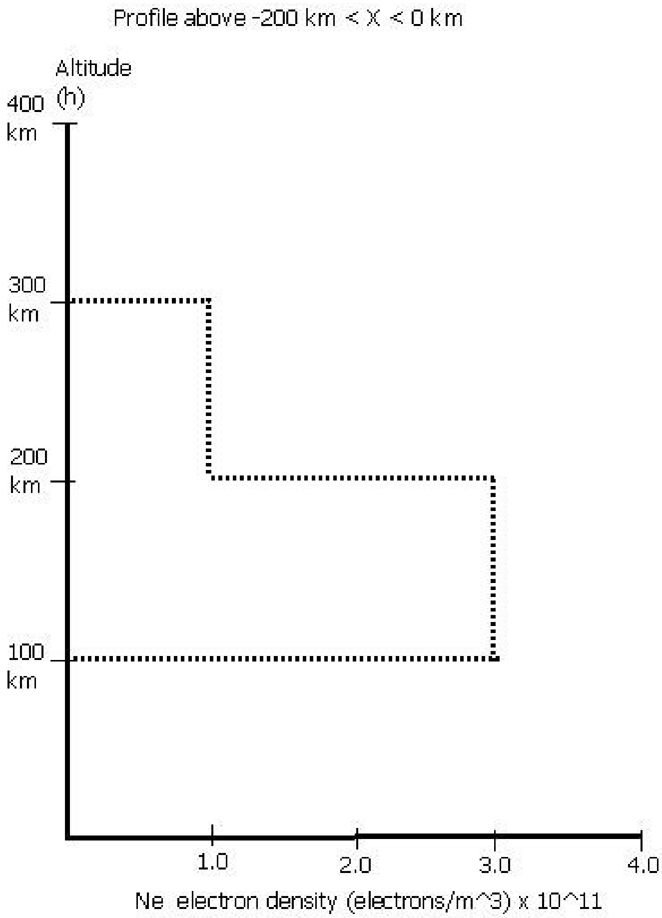
$$4.583 \times 10^{16} = 0.8333 \times 10^5 \times Ne_1 + 1.25 \times 10^5 \times Ne_3$$
$$4.124 \times 10^{16} = 1.031 \times 10^5 \times Ne_1 + 1.031 \times 10^5 \times Ne_3$$
$$6.186 \times 10^{16} = 1.031 \times 10^5 \times Ne_2 + 1.031 \times 10^5 \times Ne_4$$
$$6.666 \times 10^{16} = 0.8333 \times 10^5 \times Ne_2 + 1.25 \times 10^5 \times Ne_4$$

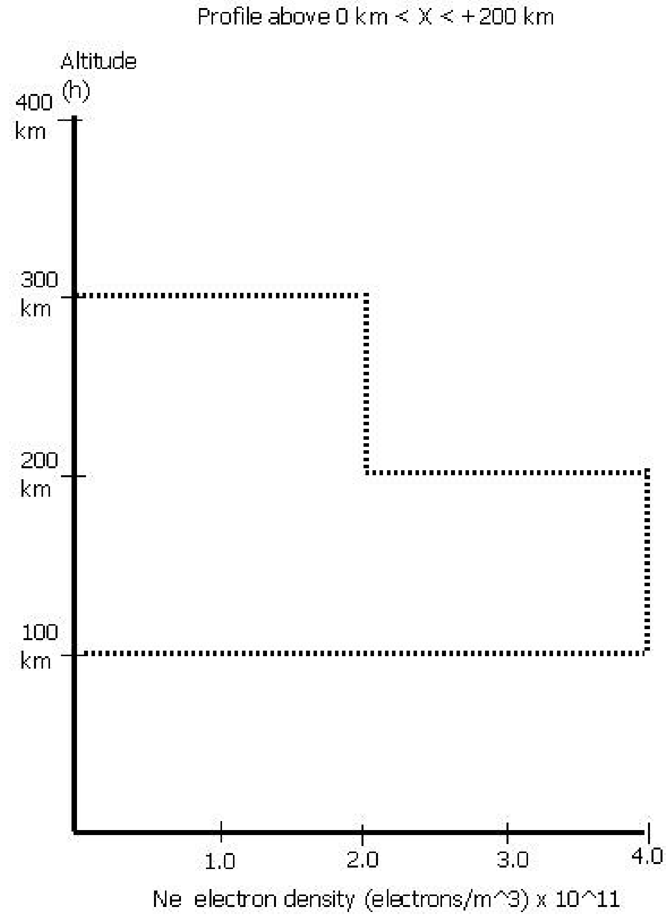
Solving, we get:

$Ne_1 = 1 \times 10^{11}$ electrons/$m^3$
$Ne_2 = 2 \times 10^{11}$ electrons/$m^3$
$Ne_3 = 3 \times 10^{11}$ electrons/$m^3$
$Ne_4 = 4 \times 10^{11}$ electrons/$m^3$

From these values we can obtain the vertical electron density profiles shown in Figure 2. Two profiles are shown: one for the region -200 km < X < 0 km and the other for the region 0 km < X < +200 km.

Figure 2: Vertical electron density profiles for 2 regions:
-200 km < X < 0 km and 0 km < X < +200 km




Perhaps this exercise has piqued your interest in the ionosphere, its measurement, and radio propagation. This link gives an overview:

https://space.fmi.fi/MIRACLE/Geotrim/Theory.html

With more detail here:

https://www.researchgate.net/publication/279412186_Medium-scale_4-D_ionospheric_tomography_using_a_dense_GPS_network

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Thanks to the GNSS Research Group at the Royal Observatory of Belgium whose animated website (http://gnss.be/ionosphere_tutorial.php#x2-70000) inspired me to put together this exercise in applied math – R.R.P.

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